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CROSS SECTIONAL CONSTANTS AND STRESS DISTRIBUTIONS OF THIN-WALLED SECTIONS

Thomas S.Z. Hu

Defence Research Establishment Atlantic

Centre de Recherches pour la Défense Atlantique

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Abstract

The equivalent beam model is widely used for predicting strength and vibration of a ship hull in a preliminary analysis. It can also be used for checking results in a large finite element model and for parametric studies of ship behaviour. This method treats a ship hull as a series of prismatic segments connected together. Each segment has its sectional properties, real and virtual masses. The program SCRAP was developed at Defence Research Establishment Atlantic for calculation of cross sectional constants and estimation of mass properties. It prepares input data files for the finite element programs VAST and TORSON and interprets the analytical results. SCRAP can be only used for some specific sections at the present time. For arbitrarily oriented sections it may give an incorrect shear centre and warping constant and thus the wrong stress distributions.

This report presents the mathematical derivations of the equations used for the calculation of cross sectional constants and stress distributions of thin-walled sections. As an improvement over the current SCRAP program, these equations are applicable to any shape of cross section, both open and closed, and are independent of the orientation of the cross section. A computer-oriented step-by-step procedure based on these equations is outlined. Several examples are also presented to verify the procedure.

Résumé

Dans une analyse préliminaire, le modèle de poutre équivalente est largement utilisé pour la prévision de la résistance et des vibrations d'une coque de navire. Il peut aussi être utilisé pour vérifier les résultats dans un modèle à éléments finis de grande dimension et dans des études paramétriques sur la tenue des navires. Dans la présente méthode, on considère qu'une coque de navire est constituée d'une série de segments prismatiques reliés ensemble. Chaque segment possède ses propriétés de section, sa masse réelle et sa masse virtuelle. Le programme SCRAP a été écrit au Centre de recherches pour la défense (Atlantique) en vue du calcul des constantes de section transversale et de l'estimation des propriétés de masse. Il prépare les fichiers d'entrée pour les programmes à éléments finis VAST et TORSON et il interprète les résultats analytiques. SCRAP ne peut pour le moment être utilisé que pour certaines sections particulières. Dans le cas de sections d'orientation arbitraire, il peut donner une constante de centre de cisaillement et de gauchissement erronée et, par conséquent, des distributions de contraintes erronées.

Ce rapport montre comment on a déduit les équations mathématiques utilisées pour calculer les constantes de section transversale et établir les distributions de contraintes des sections à pario mince. Ces équations constituent un perfectionnement par rapport au programme SCRAP actuel : elles peuvent être appliquées à toutes les formes de section transversale, ouvertes et fermées, et elles sont indépendantes de l'orientation de la section transeversale. On décrit une méthode pas à pas mécanisée basée sur ces équations. Plusieure exemples permettant de vérifier la méthode sont aussi présentés.

Contents

List of Figures

Notation

A	area of closed section
A_s	cross section area
b	end of curvilinear coordinate
b_i	width of segment i
E	Young's modulus
F	function
G	shear modulus
I_x, I_y	moments of inertia along x and y axes respectively
I_{xy}	product of inertia
I_ω	warping constant
J	St-Venant torsional constant
k	segment number
L_k	length of segment
N	axial force
M_x, M_y	moment along x and y axes respectively
M_ω	bimoment
Q_x, Q_y	shear forces along x and y axes respectively
q_b, q_s, q_ω	transverse, St-Venant and warping shear flows
q_b^*, q_ω^*	shear flows at beginnings of integration
q_b^o, q_ω^o	shear flows of open sections
S_x, S_y, S_ω	static moments of a portion of cross section in x, y and ω coordinates

s	curvilinear coordinate along wall profile
T_s, T_ω	St-Venant, warping torsion moments
t	thickness of wall
X_c	centroid in X direction
x_p	centre of rotation in x direction
Y_c	centroid in Y direction
y_p	centre of rotation in y direction
w	displacement in z direction
α	angle between tangent to s and 0-x axis
γ	shear strain
η	displacement in x direction
ξ	displacement in y direction
σ_b, σ_ω	bending and warping normal stresses
υ	displacement in s axis
Ω_c, Ω_o Ω_1, Ω_2	quantities for calculating sectoriai coordinate
ω	sectorial coordinate
φ	angle of rotation

1. Introduction

The equivalent beam model is a simplified method widely used for predicting strength and vibration of a ship hull in preliminary design. It can also be used for parametric studies and for checking results in a large finite element model. The equivalent beam method treats a ship hull as a series of prismatic segments connected together. Each segment has its own sectional properties, real and virtual masses. By utilizing different types of beam elements and numerical methods, stress distributions due to various loadings and natural frequencies of the structures can be obtained.

The various beam elements in the finite element method are derived from different assumptions, with or without shear deformation and/or longitudinal warping. For a ship with large openings, such as a container ship, the lowest natural frequency of coupled horizontal-torsional modes may be close to the lowest natural frequency of flexural vibration modes, and the horizontal propeller forces may generate large torsional moment. The shear deformation of the cross section of a ship hull also can be significant depending on the depth to length ratio of the ship. Thus, the required sectional properties for the subsequent numerical analysis are not only the moments of inertia and centroid, but also the shear centre and torsion and warping constants.

The program SCRAP was developed by the Structural Mechanics Group of Defence Research Establishment Atlantic (DREA) for calculating different sectional constants, mass properties, preparing input data files for the finite element programs VAST [1] and TORSON [2] and interpreting the analytical results. The required sectional properties for the general beam element in the finite element program VAST are the moments of inertia, the torsion constant and the shear centre. An additional sectional constant needed for the program TORSON is the warping constant. SCRAP is documented in References 3 to 6.

The program SCRAP is efficient and user friendly but can only be used for symmetric sections at the present time. It may give the incorrect shear centre and warping constant for an arbitrarily oriented section and, consequently, invalid results for the equivalent beam analyses. Because this program is unable to provide the correct shear centre for arbitrarily oriented sections, the distribution of transverse shear flows is also incorrect.

The available literature on the subject of calculating sectional constants and stress distributions of arbitrarily oriented thin-walled sections tends to be incomplete and problem specific [7,8]. This report presents complete mathematical derivations of the equations used for the calculation of the cross sectional constants and stress distributions of thin-walled sections. These equations can be used for any shape of cross section, both open and closed, and are independent of the orientation of the cross section. As well, a computer oriented step-by-step procedure based on these equations is outlined. Several examples are also given in the final section of this report to verify this procedure.

2. Derivations

Geometric discontinuities exist at large hatch openings of ship hulls and on the locations where open and closed sections mix. They create relative restraints and cause displacement incompatibilities between segments under applied torsion loading, thus generating secondary stresses in addition to the primary shear stresses and altering the overall stress distribution. This section presents the mathematical derivations of torsion induced displacements and stresses of thin-walled sections.

Several essential assumptions made herein for all of the derivations follow:

1. The beam segment is a prismatic thin-walled section. The term "thin-walled section" indicates that the thickness of the wall 't' is small in comparison with the total width of the cross section but it has sufficient thickness so that local buckling is not a problem. Thus, the shear stresses can be assumed constant through the thickness of the wall but may vary along the cross section.

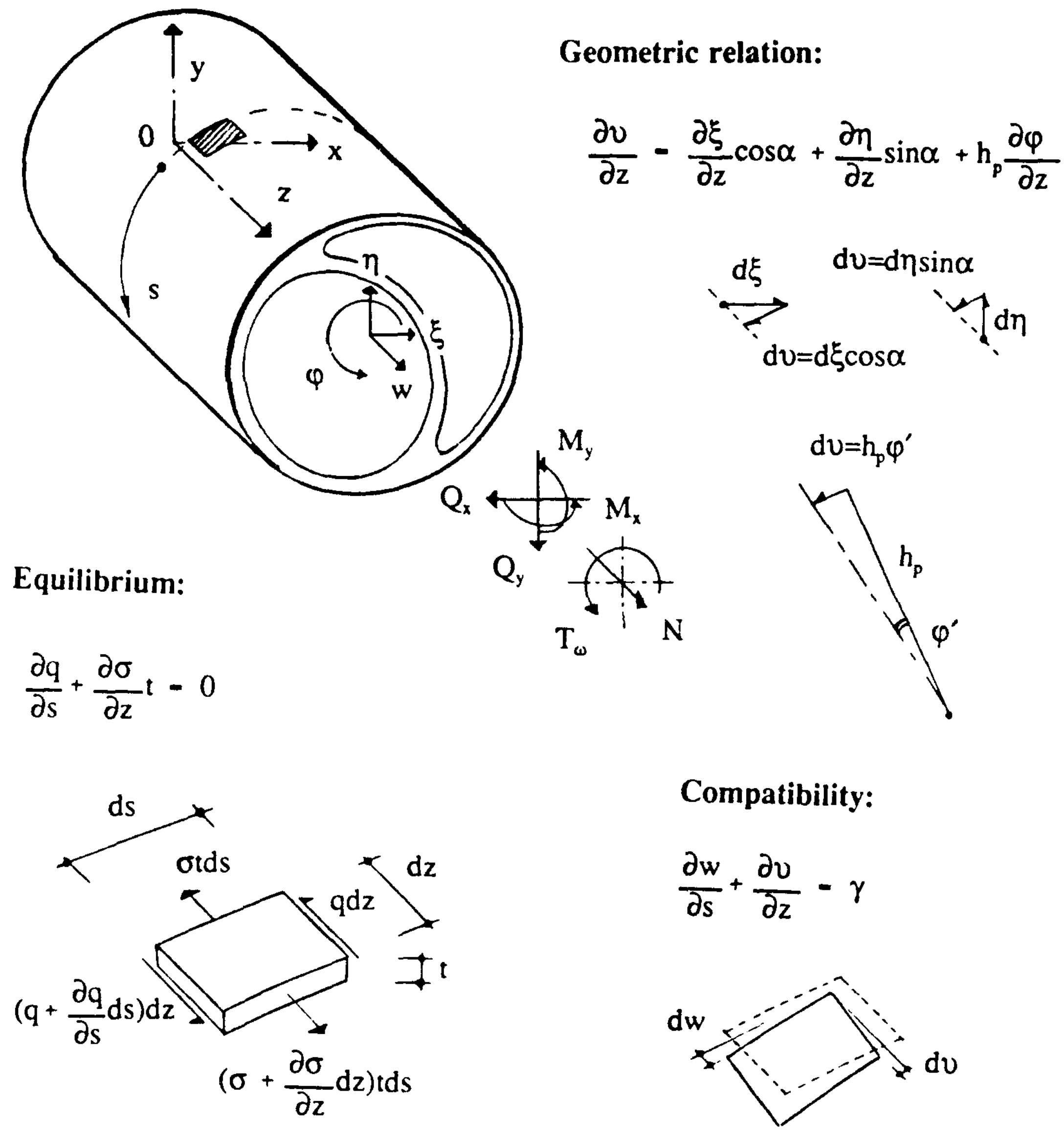
2. There is no transverse deformation occurring under applied loads. The shape of the cross section always remains unchanged.

3. The shear deformation of the cross section is caused by the primary (St-Venant) shear stresses only. The additional shear deformation caused by the secondary (warping) shear stresses can be neglected. For an open thin walled section, the net St-Venant shear stresses are equal to zero through the wall thickness; consequently, there is no shear deformation.

The sign conventions and coordinate systems of a thin-walled cross section subjected to bending and torsion for the following theoretical development are illustrated in Fig. 1. Two coordinate systems are defined; a Cartesian coordinate system which has x, y and z axes through the centre of gravity of the cross section and a curvilinear coordinate system, s, which coincides with centreline of the wall of the cross section and is bounded with boundaries s=0 and s=b. The Cartesian coordinate system has displacements ξ, η and w along the x, y and z axes and a rotation φ around the z axis. The curvilinear coordinate system, on the other hand, has a displacement υ along the s axis. The positive directions of forces and displacements are indicated by arrow heads. One should keep in mind that the sign conventions used here are different from Timoshenko's definitions [9].

2.1 General Considerations

A small wall element cut from the thin wall beam as shown in Fig. 1 has an angle α between the s and x axes. The displacements between two coordinate systems can be related to each other through geometric orientation as:



Note: α is the angle between the 0-x axis and the positive tangential direction of coordinate, s, measured counterclockwise

Figure 1: Sign conventions and coordinate systems

$$\frac{\partial \upsilon}{\partial z} = \frac{\partial \xi}{\partial z}\cos\alpha + \frac{\partial \eta}{\partial z}\sin\alpha + h_p\frac{\partial \varphi}{\partial z} \tag{1}$$

in which h_p is the distance between the tangent of a wall element to the centre of rotation 'p' of the cross section (it can be proved that the centre of rotation is also the shear centre of the cross section). The non-dependencies of α and h_p from the z axis in this equation indicate that the beam is prismatic.

The equilibrium equations between internal resultants and external forces of a cross section are:

$$\begin{aligned}
&\int_{A_s}\sigma\, dA_s = N \quad (a); &&\int_s q h_p ds = T_p \quad (d)\\
&\int_{A_s}\sigma\, x dA_s = M_x \quad (b); &&\int_s q\cos\alpha\, ds = Q_x \quad (e)\\
&\int_{A_s}\sigma\, y dA_s = M_y \quad (c); &&\int_s q\sin\alpha\, ds = Q_y \quad (f)
\end{aligned} \tag{2}$$

where A_s is the cross section area. For a small wall element as shown in Fig. 1, the equilibrium equation in the longitudinal direction of the beam is

$$\frac{\partial q}{\partial s} + \frac{\partial \sigma}{\partial z}t = 0 \tag{3}$$

where q and t are shear flow and wall thickness respectively.

The compatibility relation between displacement υ along the s axis and w along the z axis can be obtained through shear strain γ as:

$$\frac{\partial w}{\partial s} + \frac{\partial \upsilon}{\partial z} = \gamma \tag{4}$$

The constitutive relation of the material follows Hooke's law (linear elastic material) as

$$\gamma = \frac{\tau}{G} \tag{5}$$

where τ and G are shear stress and shear modulus respectively.

2.2 Pure Bending

It is convenient to describe the geometry of a cross section in an arbitrary

Cartesian coordinate system (X,Y). The arbitrary coordinate system's origin does not necessarily coincide with the centre of gravity of the cross section but its axes are parallel to the cross section's x and y axes so that dX=dx=ds cosα and dY=dy=ds sinα. The transformation between the two Cartesian coordinate systems can be obtained through the calculation of the centre of gravity as:

$$X_c = \frac{\int_{A_s} X dA_s}{A_s}; \qquad x = X - X_c$$
$$Y_c = \frac{\int_{A_s} Y dA_s}{A_s}; \qquad y = Y - Y_c \tag{6}$$

By assuming no shear deformation caused by transverse shear (Bernoulli-Naviers hypothesis), using equations (1) and (3) followed by integration along the s axis, the longitudinal displacement w can be expressed as

$$w = -\xi' X - \eta' Y + w_o(z) \tag{7}$$

where $w_o(z)$ is the longitudinal displacement in the origin of the system (X,Y) and is an unknown.

Through the constitutive relation, the axial stress caused by bending can be obtained as:

$$\sigma_b = -E\xi'' X - E\eta'' Y + E w_o'(z) \tag{8}$$

The term w_o can be eliminated by knowing the fact that there is no axial resultant for a beam subjected to pure bending. Letting the axial force N in equation (2a) equal zero and using the coordinate transformation in equation (6), yields:

$$\sigma_b = -E\xi'' x - E\eta'' y \tag{9}$$

The moments of inertia of a plane section with respect to the x and y axes are defined by the integrals:

$$I_x = \int_{A_s} x^2 dA_s; \qquad I_y = \int_{A_s} y^2 dA_s; \qquad I_{xy} = \int_{A_s} xy dA_s \tag{10}$$

The moment curvature relationships of a prismatic beam are expressed as:

$$
\begin{aligned}
-E\xi''\int_{A_s} x^2 dA_s - E\eta''\int_{A_s} xy dA_s &= M_x \\
-E\xi''\int_{A_s} xy dA_s - E\eta''\int_{A_s} y^2 dA_s &= M_y
\end{aligned}
\tag{11}
$$

By substituting these quantities into equation (9), the axial stress can be rewritten as:

$$
\sigma_b = \frac{I_y x - I_{xy} y}{D} M_x + \frac{I_x y - I_{xy} x}{D} M_y \tag{12}
$$

where $D = I_x I_y - I_{xy}^2$.

The integration of equation (2) along the curvilinear coordinate leads to

$$
q_b(s,z) = q_b^*(z) - \int_0^s \frac{\partial \sigma}{\partial z} t ds \tag{13}
$$

where the constant q_b^* represents the shear flow at the beginning of the integration. The shear flow, q_b^* vanishes for an open section with the integration starting at a free edge (shear flow equals zero at this point) and is unknown for a closed section in which the shear flow of the starting integration point usually is non-zero.

Introducing two integrals which represent static moments of a portion of the cross section as:

$$
S_x = \int_0^s x t ds; \qquad S_y = \int_0^s y t ds \tag{14}
$$

and substituting equation (12) into equation (13), one can obtain

$$
q_b = q_b^* - \frac{I_y S_x - I_{xy} S_y}{D} Q_x - \frac{I_x S_y - I_{xy} S_x}{D} Q_y \tag{15}
$$

By defining two quantities

$$
q_{bx}^{ou} = -\frac{I_y S_x - I_{xy} S_y}{D}; \qquad q_{by}^{ou} = -\frac{I_x S_y - I_{xy} S_x}{D} \tag{16}
$$

and with the fact that q_b is a linear combination of the shear forces Q_x and Q_y, equation (15) can be written as:

$$q_b = (q^{*}_{bx} - \frac{I_y S_x - I_{xy} S_y}{D} Q_x) + (q^{*}_{by} - \frac{I_x S_y - I_{xy} S_x}{D} Q_y) \qquad \textbf{(17)}$$
$$= (q^{*u}_{bx} + q^{ou}_{bx}) Q_x + (q^{*u}_{by} + q^{ou}_{by}) Q_y$$

where superscript "u" can be interpreted as shear flow caused by a unit of shear force.

Since there is no twisting, the unit angle of twist is zero which indicates that

$$\int_o^b \frac{q_b}{t} ds = 0 \qquad \textbf{(18)}$$

Two unknown quantities q_{bx}^{*u} and q_{by}^{*u} can be obtained as

$$q^{*u}_{bx} = - \frac{\int_o^b \frac{q^{ou}_{bx}}{t} ds}{\int_o^b \frac{ds}{t}} ; \quad q^{*u}_{by} = - \frac{\int_o^b \frac{q^{ou}_{by}}{t} ds}{\int_o^b \frac{ds}{t}} \qquad \textbf{(19)}$$

They are equal to zero for an open section. On the other hand, for a multi-cell section with n compartments these equations have to be satisfied for each cell and thus result in n simultaneous linear equations.

The moments of inertia in equations (12) and (15), based on the assumption of a thin walled section, can be simplified by using integration by parts as:

$$I_x = \int_{A_s} x^2 dA_s = \int_o^b x^2 t ds = x S_x |_o^b - \int_o^b S_x dx \qquad \textbf{(20)}$$

Because the first term on the right hand side of the equation vanishes (according to equation (6), x is normalized so $S_x(0)$ and $S_x(b)$ equal zero), the moments of inertia can be expressed as

$$I_x = - \int_o^b S_x dx ; \quad I_y = - \int_o^b S_y dy ; \quad I_{xy} = - \int_o^b S_x dy = - \int_o^b S_y dx \qquad \textbf{(21)}$$

2.3 Twisting

There are no lateral displacements, ξ and η, for a beam subjected only to pure twisting around its centre of rotation. Thus, equation (3) can be reduced to

$$\frac{\partial \upsilon}{\partial z} = \varphi' h_p \qquad \textbf{(22)}$$

According to the basic assumption that the shear deformation of the cross section is only caused by the St-Venant shear stresses, with equation (4), yields

$$\begin{aligned}\frac{\partial w}{\partial s} &= -h_p \varphi' + \frac{q_s}{Gt} \\ &= -h_p \varphi' + \frac{J}{2At}\varphi'\end{aligned} \qquad \textbf{(23)}$$

where A is the area of a closed section surrounded by the thin walls. The second term in this equation vanishes for an open section.

By defining $d\Omega_1 = h_p ds$ and $d\Omega_2 = J/(2At)ds$, integrating equation (23) along the s axis, gives

$$w = -\varphi'\Omega_1 + \varphi'\Omega_2 + w_o(z) \qquad \textbf{(24)}$$

The warping normal stresses can be obtained by using Hooke's law derived as

$$\sigma_\omega = -E\varphi''\Omega_1 + E\varphi''\Omega_2 + Ew_o'(z) \qquad \textbf{(25)}$$

Letting $w_o'/\varphi''=\Omega_o$, and following Kollbrunner's [7] definition with a normalized sectorial coordinate, will lead to

$$\omega = \Omega_1 - \Omega_2 - \Omega_o \qquad \textbf{(26)}$$

allowing the axial stress in equation (25) to be simplified as

$$\sigma = -E\varphi''\omega \qquad \textbf{(27)}$$

The quantity Ω_o in equation (26) can be evaluated using the fact that the resultant axial force acting on the cross section is equal to zero. Letting N in equation (2a) equal to zero and integrating warping normal stresses along the s axis yields

$$\Omega_o = \frac{\int_o^b (\Omega_1 - \Omega_2)tds}{\int_o^b tds} \tag{28}$$

The warping shear flow can be obtained from equation (3) as

$$q_w = E\varphi''' \int_o^s \omega\, t\, dS + q_w^* \tag{29}$$

where q_ω^* represents the shear flow at the beginning of the integration point.

By introducing a sectorial static moment of a cut-off portion of the cross section,

$$S_\omega = \int_o^s \omega\, tds \tag{30}$$

the warping shear flow can be expressed as

$$\begin{aligned} q_\omega &= E\varphi''' S_\omega + q_\omega^* \\ &= q_\omega^o + q_\omega^* \end{aligned} \tag{31}$$

The warping torsional moment with respect to the point p can be calculated

$$T_\omega = \int_o^b q_\omega h_p ds = \int_o^b q_\omega \frac{\partial \omega}{\partial s} ds + \int_o^b q_\omega \frac{\partial \Omega_2}{\partial s} ds \tag{32}$$

The assumption (3) that the longitudinal shear strain is only caused by the St-Venant shear flow, and that warping shear flow has no contribution to the shear deformation, indicates that

$$\int_o^b \gamma_\omega ds = \int_o^b \frac{q_\omega}{Gt} ds = 0 \tag{33}$$

Thus the last term in equation (32) is equal to zero

$$\int_o^b q_\omega \frac{\partial \Omega}{\partial s}{}_2 ds = \int_o^b q_\omega \frac{J}{2At} ds = \int_o^b \frac{q_\omega}{Gt} \frac{\tau_t t}{\varphi'} ds = 0 \tag{34}$$

Using integration by parts of the first term in equation (32), the warping torsional moment can be obtained as

$$\begin{aligned} T_\omega &= q_\omega \omega \,|_o^b - \int_o^b \omega \frac{\partial q_\omega}{\partial s} ds \\ &= +\int_o^b \frac{\partial \sigma}{\partial z} \omega \, tds = -E\varphi''' \int_o^b \omega^2 tds \end{aligned} \tag{35}$$

or

$$T_\omega = -E\varphi''' I_\omega \tag{36}$$

where I_ω is the warping constant

$$I_\omega = \int_o^b \omega^2 tds \tag{37}$$

The warping shear flow can be expressed as

$$\begin{aligned} q_\omega &= -\frac{T_\omega}{I_\omega} S_\omega + q_\omega^{*} \\ &= (q_\omega^{ou} + q_\omega^{*u}) T_\omega \end{aligned} \tag{38}$$

The constant shear flow q_ω^{*u} caused by a unit warping moment can be obtained according to assumption (3) or equation (33) as

$$q_\omega^{*u} = -\frac{\int_o^b \frac{q_\omega^{ou}}{t} ds}{\int_o^b \frac{ds}{t}} \tag{39}$$

Defining $M_\omega' = T_\omega$, the warping normal stress can be expressed as

$$\sigma = \frac{M_\omega}{I_\omega}\omega \tag{40}$$

The derivation of the above equations requires that the centre of rotation is known in advance and that all the nodal coordinates refer to this point. Using geometric relationships leads to

$$h_p ds = h_c ds - x_p \sin\alpha \; ds + y_p \cos\alpha \; ds \tag{41}$$

Letting $d\Omega_c = h_c\, ds$ and using the fact that $dy = \sin\alpha\, ds$ and $dx = \cos\alpha\, ds$, the normalized sectorial coordinate, ω, can be expressed as

$$\omega = \Omega_c - x_p y + y_p x - \Omega_2 - \Omega_o \tag{42}$$

The centre of rotation can be evaluated by using the fact that there are no flexural moments under pure twisting. Letting equations (2b) and (2c) equal zero gives

$$\begin{aligned} &\int_o^b \sigma\, xtds = 0; \quad \int_o^b \sigma\, ytds = 0 \\ \text{or} \quad &\int_o^b \omega\, xtds = 0; \quad \int_o^b \omega\, ytds = 0 \end{aligned} \tag{43}$$

Introducing equation (42) into (43), gives

$$\begin{aligned} &\int_o^b \omega_c xtds - x_p \int_o^b yxtds + y_d \int_o^b x^2 tds - \int_o^b \Omega_2 xtds = 0 \\ &\int_o^b \omega_c ytds - x_p \int_o^b y^2 tds + y_p \int_o^b xytds - \int_o^b \Omega_2 ytds = 0 \end{aligned} \tag{44}$$

Defining

$$\begin{aligned} I_{x\Omega_c} &= \int_o^b \Omega_c xtds; \quad I_{x\Omega_2} = \int_o^b \Omega_2 xtds \\ I_{y\Omega_c} &= \int_o^b \Omega_c ytds; \quad I_{y\Omega_2} = \int_o^b \Omega_2 ytds \end{aligned} \tag{45}$$

and introducing these quantities into equation (44) and solving two simultaneous linear equations, the centre of twisting can be obtained as

$$
\begin{aligned}
x_p &= \frac{(I_{y\Omega_c} - I_{y\Omega_2})I_x - (I_{x\Omega_c} - I_{x\Omega_2})I_{xy}}{D} \\
y_p &= -\frac{(I_{x\Omega_c} - I_{x\Omega_2})I_y - (I_{y\Omega_c} - I_{y\Omega_2})I_{xy}}{D}
\end{aligned}
\qquad (46)
$$

The quantities in equation (45) can also be simplified by following the same procedure as equation (20), using integration by parts as

$$
\begin{aligned}
I_{x\Omega_c} &= -\int_o^b S_x d\Omega_c; \qquad I_{x\Omega_2} = -\int_o^b S_x d\Omega_2 \\
I_{y\Omega_c} &= -\int_o^b S_y d\Omega_2; \qquad I_{y\Omega_2} = -\int_o^b S_y d\Omega_2
\end{aligned}
\qquad (47)
$$

3. Numerical Procedure

The thin-walled section is assumed to be composed of a number of narrow rectangular segments. Each segment is numbered consecutively, starts with node i and ends with node j. The coordinates x, y and ω for each segment are distributed linearly along its length; therefore, the static moments S_x, S_y and S_ω vary parabolically along each element. A three points rule of Newton-Cotes integration can be used to obtain an exact solution for the cross sectional constants I_x, I_y, I_{xy} and I_ω. The integration of function f(r) from r=a to r=b can be evaluated as

$$
\int_a^b f(r)dr = \frac{b-a}{6}[f(a)+4f(\frac{a+b}{2})+f(b)] \qquad (48)
$$

With the equations derived in the previous section, the step-by-step procedure to evaluate the cross sectional constants is as follows:

(1) Input nodal coordinates (X_i,Y_i) and (X_j,Y_j) of each segment k with respect to an arbitrary coordinate system (X,Y), its thickness t_k and connectivity with other elements.

(2) Calculate coordinates of mid-point, length and area of each segment k
(3) Obtain centre of gravity of the cross section as

$$X_k = \frac{X_i + X_j}{2}; \qquad \Delta X_k = X_i - X_j$$

$$X_k = \frac{Y_i + Y_j}{2}; \qquad \Delta Y_k = Y_i - Y_j$$

$$L_k = \sqrt{\Delta X^2 + \Delta Y^2}; \qquad A_k = L_k t_k$$

$$X_c = \frac{\sum X_k A_k}{\sum A_k}; \qquad Y_c = \frac{\sum Y_k A_k}{\sum A_k}$$

and the nodal and mid-point coordinate of each segment (x_i, y_i), (x_j, y_j) and (x_k, y_k) with respect to the new origin (X_c, Y_c) as

$$x = X - X_c; \qquad y = Y - Y_c$$

(4) Calculate the static moments S_x and S_y of a cut of portion of the cross section for points i, j and k of each segment as

$$\Delta S_x = \frac{A_k(x_i + x_k)}{4}; \qquad S_x = S_x + \Delta S_x$$

$$\Delta S_y = \frac{A_k(y_i + y_k)}{4}; \qquad S_y = S_y + \Delta S_y$$

(5) Obtain the moments of inertia of the cross section as

$$I_x = \sum_{n=1}^{k} \frac{\Delta X_k}{6}[S_x(i) + 4S_x(k) + S_x(j)]; \qquad I_y = \sum_{n=1}^{k} \frac{\Delta Y_k}{6}[S_y(i) + 4S_y(k) + S_y(j)]$$

$$I_{xy} = \sum_{n=1}^{k} \frac{\Delta Y_k}{6}[S_x(i) + 4S_x(k) + S_x(j)]; \qquad I_{yx} = \sum_{n=1}^{k} \frac{\Delta X_k}{6}[S_y(i) + 4S_y(k) + S_y(j)]$$

(6) Calculate $\Delta\Omega_{ck}$ as

$$\Delta\Omega_{ck} = x_k \Delta Y_k - y_k \Delta X_k$$

(7) Evaluate $\Delta\Omega_{2k}$ as

$$\Delta\Omega_{2k} = 2K^{-1}A \frac{L_k}{t_k}$$

For an open section, this term is equal to zero. For a multi-cell closed section with n cells, K is an $n \times n$ squared matrix as described in Appendix (A) and A is an $n \times 1$ matrix.

(8) Evaluate $I_{x\Omega c}$, $I_{y\Omega c}$, $I_{x\Omega 2}$ and $I_{y\Omega 2}$ as

$$I_{x\Omega c} = \sum_{n=1}^{k} \frac{\Delta\Omega_{ck}}{6}[S_x(i)+4S_x(k)+S_x(j)]; \quad I_{y\Omega c} = \sum_{n=1}^{k} \frac{\Delta\Omega_{ck}}{6}[S_y(i)+4S_y(k)+S_y(j)]$$

$$I_{x\Omega 1} = \sum_{n=1}^{k} \frac{\Delta\Omega_{1k}}{6}[S_x(i)+4S_x(k)+S_x(j)]; \quad I_{y\Omega 1} = \sum_{n=1}^{k} \frac{\Delta\Omega_{1k}}{6}[S_y(i)+4S_y(k)+S_y(j)]$$

and centre of rotation of the cross section $(x_p, y_p\}$ as

$$x_p = \frac{(I_{y\Omega_c} - I_{y\Omega_2})I_x - (I_{x\Omega_c} - I_{x\Omega_2})I_{xy}}{D}; \quad y_p = -\frac{(I_{x\Omega_c} - I_{x\Omega_2})I_y - (I_{y\Omega_c} - I_{y\Omega_2})I_{xy}}{D}$$

(9) Calculate $\Delta\Omega_{1k}$ as

$$\Delta\Omega_{1k} = \Delta\Omega_{ck} - x_p\Delta Y_k + y_p\Delta X_k$$

(10) Calculate

$$\Delta(\Omega_{1k} - \Omega_{2k}) = \Delta\Omega_{1k} - \Delta\Omega_{2k}$$

$$(\Omega_1 - \Omega_2)_i = (\Omega_1 + \Omega_2)_{i-1} + \Delta(\Omega_1 + \Omega_2)$$

$$\Omega_o = \frac{\sum_{n=1}^{k}[((\Omega_1+\Omega_2)_i+(\Omega_1+\Omega_2)_j)\frac{A_k}{2}]}{\sum_{n=1}^{k} A_k}$$

(11) Calculate normalized sectorial coordinate of nodal and mid-point of each segment

$$\omega_i = \Omega_1 - \Omega_2 - \Omega_o$$

(12) Calculate S_ω of a cut-off portion for each segment as

$$\Delta S_\omega = \frac{A_k(\omega_i + \omega_k)}{4}; \quad S_\omega = S_\omega + \Delta S_\omega$$

(13) Obtain the warping constant I_ω as

$$I_\omega = \sum_{n=1}^{k} \frac{\Delta\Omega_1 - \Delta\Omega_2}{6}[S_\omega(i) + 4S_\omega(k) + S_\omega(j)]$$

The calculations of normal stresses are straight forward and do not need to be discussed. Only the procedure for calculating the transverse and the warping shear stresses are presented here:

(14) Calculate q_{bx}^{ou} and q_{by}^{ou} as

$$q_{bx}^{ou} = -\frac{I_y S_x - I_{xy} S_y}{D}; \qquad q_{by}^{ou} = -\frac{I_x S_y - I_{xy} S_x}{D}$$

(15) Calculate the total shear flow around each cell as

$$(q_{bx}^{ou})_{cell} = [\sum \frac{L_k}{t_k}[q_{bx}^{ou}(i) + 4q_{bx}^{ou}(k) + q_{bx}^{ou}(j)]]_{cell}$$
$$(q_{by}^{ou})_{cell} = [\sum \frac{L_k}{t_k}[q_{by}^{ou}(i) + 4q_{by}^{ou}(k) + q_{by}^{ou}(j)]]_{cell}$$

and the two quantities q_{bx}^{*u} and q_{by}^{*u} as

$$\{q_{bx}^{*u}\} = -[K]^{-1}\{(q_{bx}^{ou})_{cell}\}$$
$$\{q_{by}^{*u}\} = -[K]^{-1}\{(q_{by}^{ou})_{cell}\}$$

(16) The transverse shear flow caused by a unit shear force can be obtained as

$$q_{bx}^{u} = q_{bx}^{ou} + q_{bx}^{*u}$$
$$q_{by}^{u} = q_{by}^{ou} + q_{by}^{*u}$$

(17) Calculate q_ω^{ou} as

$$q_\omega^{ou} = -\frac{S_\omega}{I_\omega}$$

(18) Calculate the total shear flow around each cell as

$$(q_\omega^{ou})_{cell} = [\sum \frac{L_k}{t_k}[q_\omega^{ou}(i) + 4q_\omega^{ou}(k) + q_\omega^{ou}(j)]]_{cell}$$

and q_ω^{*u} as

$$\{q_\omega^{*u}\} = -[K]^{-1}\{(q_\omega^{ou})_{cell}\}$$

(19) The warping shear flow caused by a unit shear force can be obtained as

$$q_\omega^{u} = q_\omega^{ou} + q_\omega^{*u}$$

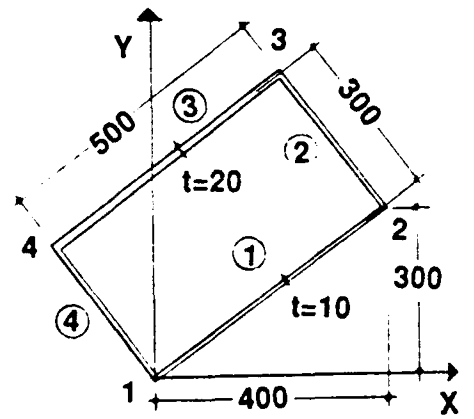
(19) The shear flows caused by different forces now can be evaluated by multiplying these unit shear flows with applied forces.

The above procedure can be applied on not only the closed sections but also the open sections by letting the quantities Ω_2 and q^* equal zero.

4. Examples

Two examples are presented in this section in tabular form in order to demonstrate the above procedure. The Arabic numeral in the first row of each table indicates the step number. The results can be verified with the equilibrium condition.

Example 1: A single-cell box beam with different wall thicknesses is shown in figure 2. (a) Calculate the cross sectional constants and shear centre. (b) Calculate the transverse shear flow caused by transverse shear forces Q_x=-5.5 × 10^5 kN and Q_y=-4.125 × 10^5 kN. (c) Calculate the warping shear flow due to a unit warping moment.



$[K] = 135$ $\qquad$ $\{A\} = \{15\times10^4\}$

$X_c = 88.6$ $\qquad$ $Y_c = 298.6$

$x_p = -10.32$ $\qquad$ $y_p = -13.83$

$X_p = 78.28$ $\qquad$ $Y_p = 312.43$

$I_x = 56.84 \times 10^7$

$I_y = 47.49 \times 10^7$ $\qquad$ $J = 6.67 \times 10^8$

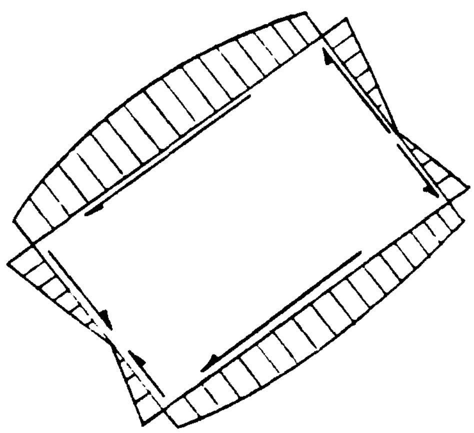
$I_{xy} = 15.91 \times 10^7$

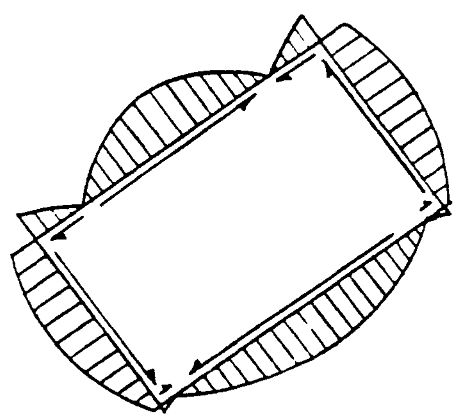
$I_\omega = 11.71 \times 10^{10}$

$\{q_{bx}^{ou}\} = \{-15372 \times 10^{-5}\}$

$\{q_{by}^{ou}\} = \{10369 \times 10^{-5}\}$

$\{q_\omega^{ou}\} = \{-96.37 \times 10^{-5}\}$





Transverse shear flow $\qquad$ **Warping shear flow**

(Normalized)

Figure 2: Single-cell box beam

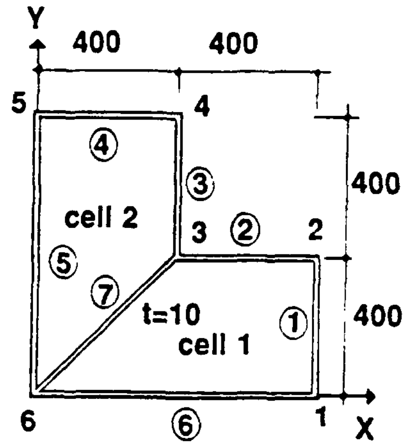
			(1)			(2)		(3)		(4)			(5)	(6)	(7)		(8)
elem	t_k	node	X	ΔX	L_k	A_k 10^3	X_kA_k 10^5	x	ΔS_X 10^5	S_X	1+4+1 10^5	I_X 10^7	I_{XY} 10^7	$\Delta\Omega_{ck}$	$\Delta\Omega_{2k}$	$I_{X\Omega c}$ 10^{10}	$I_{X\Omega 2}$ 10^{10}
		1	0					-88.6		0							
									0.29								
1	10		200	400	500	5	10	111.4		0.29	6.74	26.96	20.22	92860	111111	6.26	7.49
									5.29								
		2	400					311.4		5.58							
									4.0								
2	10		310	-180	300	3	9.3	221.4		9.58	56.13	-101.03	134.71	74988	66666	42.09	37.42
									2.65								
		3	220					131.4		12.23							
									1.57								
3	10		20	-400	500	10	2	-68.6		13.8	72.8	-291.2	-218.4	57140	55556	41.6	40.44
									-8.43								
		4	-180					-268.6		5.37							
									-3.35								
4	20		-90	180	300	3	-2.7	-178.6		2.02	13.46	24.21	-32.28	75012	66667	10.09	8.97
									-2.0								
		1	0					-88.6		0							
Σ						21	18.6					-341.06	-95.75			100.04	94.32
							88.6				-Σ/6	56.84	15.91			-16.67	-15.72

			(1)			(2)		(3)		(4)			(5)		(8)
elem	t_k	node	Y	ΔY	L_k	A_k 10^3	Y_kA_k 10^5	y	ΔS_Y 10^5	S_Y	1+4+1 10^5	I_Y 10^7	I_{YX} 10^7	$I_{Y\Omega c}$ 10^{10}	$I_{Y\Omega 2}$ 10^{10}
		1	0					-298.6		0					
									-5.59						
1	10		150	300	500	5	7.5	-148.6		-5.59	-29.79	-89.37	-119.16	-27.66	-33.1
									-1.84						
		2	300					1.4		-7.43					
									0.92						
2	10		420	240	300	3	12.6	121.4		-6.51	-37.26	-89.42	67.07	-27.94	-24.84
									2.72						
		3	540					241.4		-3.79					
									8.32						
3	10		390	-300	500	10	39	91.4		4.53	19.68	-59.04	-78.72	11.24	10.93
									0.82						
		4	240					-58.6		5.35					
									-1.78						
4	20		120	-240	300	3	3.6	-178.6		3.57	19.63	-47.11	35.33	14.72	13.09
									-3.56						
		1	0					-298.6		0					
Σ						21	62.7					-284.94	-95.48	-29.64	-33.92
							298.6				-Σ/6	47.49	15.91	4.94	5.65

		(1)	(2)	(9)			(10)	(11)		(12)		(13)
elem	t_k	node	A_k 10^3	$\Delta\Omega_{1k}$	$\Delta(\Omega_{1k}-\Omega_{2k})$	$\Omega_1-\Omega_2$	Ω_o	ω	ΔS_ω 10^5	S_ω 10^5	1+4+1 10^5	I_ω 10^{10}
		1				0		4830		0		
									60.38			
1	10		5	101450	-9661	-4830.5	-241.53	0		60.38	241.5	-23.3
									-60.38			
		2				-9661		-4830		0		
									-41.21			
2	10		3	75000	8334	-5494	-164.82	-664		-41.21	-184.73	-15.4
									21.29			
		3				-1327		3503		-19.92		
									87.58			
3	20		10	48550	-7006	-4830	-483	0		67.66	230.83	-16.17
									-87.58			
		4				-8333		-3503		-19.92		
									-21.29			
4	10		3	75000	8333	-4166.5	-125	664		-41.21	-184.73	-15.39
									-41.21			
		1				0		4830		0		
							-1014.35					-70.26
							-4830					11.71

	(1)	(2)	(14)	(15)	(16)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
elem	L_k	L_k/t_k	q_{bx}^{**} 10^{-5}	1+4+1 10^{5}	q_{bx}^{*} 10^{5}	q_{by}^{**} 10^{-5}	1+4+1 10^{-5}	q_{by}^{*} 10^{-5}	q_{ω}^{**} 10^{-5}	1+4+1 10^{5}	q_{ω}^{*} 10^{5}	q_b kN/mm
			0		113.9	0		-76.8	0		0.71	-308
1	500	50	-41.9	-323.9	72	128.1	720.4	51.3	-5.16	-20.64	-4.45	-607
			-156.3		-42.4	208		131.2	0		0.71	-308
2	300	30	-228.3	-1331.1	-114.4	213	1227.5	136.2	3.52	15.78	4.23	67
			-261.6		-147.7	167.5		90.7	1.7		2.41	438
3	500	25	-238.6	-1285.2	-124.7	-15.3	17.2	-92.1	-5.78	-19.72	-5.07	1066
			-69.2		44.7	-89.1		-165.9	1.7		2.41	438
4	300	30	-15.8	-132.4	98.1	-69.9	-368.7	-146.7	3.52	15.78	4.23	66
			0		113.9	0		-76.8	0		0.71	-308
		135										
				113.9			-76.8			0.71		

Example 2: For a two-cell section with constant wall thicknesses as shown figure 3, calculate (a) the cross sectional constants and shear centre, (b) the transverse shear flow caused by Q_x and Q_y equal to 10^5 kN, and (c) the warping shear flow caused by a unit warping moment.



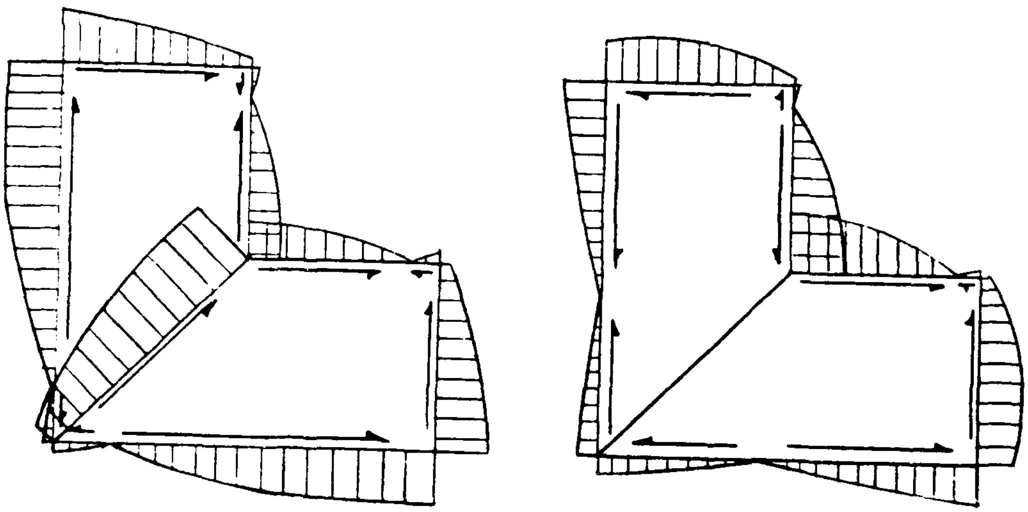
$$[K] = \begin{bmatrix} 216.56 & -56.56 \\ -56.56 & 216.56 \end{bmatrix} \quad \{A\} = \begin{Bmatrix} 24\times10^4 \\ 24\times10^4 \end{Bmatrix}$$

$I_x = 287.66 \times 10^7$
$I_y = 287.66 \times 10^7$
$I_{xy} = -53.69 \times 10^7$
$I_\omega = 2453.36 \times 10^{10}$
$J = 2.88 \times 10^9$

$$\{q_{bx}^{ou}\} = \{q_{by}^{ou}\} = \begin{Bmatrix} -2987\times10^{-5} \\ -15887\times10^{-5} \end{Bmatrix}$$

$X_c = 327.5$ $\quad Y_c = 327.5$
$x_p = -65.04$ $\quad y_p = -65.04$
$X_p = 262.46$ $\quad Y_p = 262.46$

$$\{q_\omega^{ou}\} = \begin{Bmatrix} -5217.33\times10^{-5} \\ -5217.33\times10^{-5} \end{Bmatrix}$$



(Normalized)

Transverse shear flow $\qquad$ Warping shear flow

Figure 3: Multi-cell section

			(1)			(2)		(3)		(4)			(5)	(6)	(7)		(8)
elem	t_s	node	X	ΔX	L_s	A_s 10^3	X_sA_s 10^5	x	ΔS_x 10^5	S_x 10^5	1+4+1 10^5	I_x 10^7	I_{XY} 10^7	$\Delta\Omega_s$	$\Delta\Omega_s$	$L_{s\Omega}$ 10^{10}	$L_{s\Omega 2}$ 10^{10}
		1	800					472.5		0							
									9.45								
1	10		800	0	400	4	32	472.5		9.45	56.7	0	226.8	189016	120000	107.2	68
									9.45								
		2	800					472.5		18.9							
									7.45								
2	10		600	-400	400	4	24	272.5		26.35	154.1	-616.4	0	29016	120000	44.7	184.93
									3.45								
		3	400					72.5		29.8							
									1.45								
3	10		400	0	400	4	16	72.5		31.25	187.5	0	750	29016	120000	54.4	225.03
									1.45								
		4	400					72.5		32.7							
									-0.55								
4	10		200	-400	400	4	8	-172.5		32.15	188.9	-755.6	0	189016	120000	357.1	226.7
									-4.55								
		5	0					-327.5		27.6							
									-13.1								
5	10		0	0	800	8	0	-327.5		14.5	87	0	-696	261968	240000	228	208.92
									-13.1								
		6	0					-327.5		1.4							
										-5.81*							
									-5.1								
6	10		400	800	800	8	32	72.5		-10.9	-49.41	-395.28	0	262032	240000	-129.4	-118.54
									10.9								
		1	800					472.5		0							
		3	400					72.5		0							
									-0.78								
7	[illegible]		200	-400	565	5.7	11.4	-127.5		-0.78	-10.33	41.32	41.32	0	0	0	0
									-6.43								
		6	0					-327.5		-7.21							
Σ							123.4					-1725.96	322.12			662	794.94
							327.5				-Σ/6	287.66	-53.69			-110.3	-132.5

(1)				(2)			(3)		(4)		(5)			(8)	
elem	t_e	node	Y	ΔY	L_e	A_e 10^3	$Y_e A_e$ 10^5	y	ΔS_Y 10^5	S_Y 10^5	1+4+1 10^5	I_Y 10^7	I_{YX} 10^7	$I_{Y\omega x}$ 10^{10}	$I_{Y\omega\omega}$ 10^{10}
		1	0					-327.5		0					
									-4.55						
1	10		200	400	400	4	8	-127.5		-4.55	-23.3	-93.2	0	-44	-27.96
									-0.55						
		2	400					72.5		-5.1					
									1.45						
2	10		400	0	400	4	16	72.5		-3.65	-21.9	0	87.58	-6.4	-26.27
									1.45						
		3	400					72.5		-2.2					
									3.45						
3	10		600	400	400	4	24	272.5		1.25	11.5	46	0	3.3	13.82
									7.45						
		4	800					472.5		8.7					
									9.45						
4	10		800	0	400	4	32	472.5		18.15	108.9	0	-435.7	205.8	130.71
									9.45						
		5	800					472.5		27.6					
									10.9						
5	10		400	-800	800	8	32	72.5		38.5	215	-1720	0	563.3	516.08
									-5.1						
		6	0					-327.5		33.4					
										26.2*					
									-13.1						
6	10		0	0	800	8	0	-327.5		13.1	78.6	0	628.62	205.9	188.56
									-13.1						
		1	0					-327.5		0					
		3	400					72.5		0					
									-0.78						
7	10		200	-400	565	5.7	11.4	-127.5		-0.78	-10.33	41.32	41.32	0	0
									-6.43						
		6	0					-327.5		-7.21					
Σ						37.7	123.4					-1725.96	321.72	928	794.94
							327.5				-Σ/6	287.66	-53.69	-154.6	-132.5

		(1)	(2)	(9)			(10)	(11)		(12)		(13)
elem	l_k	node	A_k 10^3	$\Delta\Omega_{1k}$	$\Delta(\Omega_{1k}-\Omega_{2k})$	$\Omega_1-\Omega_2$	Ω_k 10^3	ω	ΔS_ω 10^3	S_ω 10^3	1+4+1 10^3	I_ω 10^{10}
		1				0		-30064		0		
									-126.1			
1	10		4	215032	95032	47516	1900.64	17452		-126.1	193.6	183.9
									824.2			
		2				95032		64968		698.1		
									974.5			
2	10		4	55032	-64968	62548	2501.92	32484		1672.6	9385.9	-6097.8
									324.8			
		3				30064		0		1997.4		
									-324.8			
3	10		4	55032	-64968	-2420	-96.8	-32484		1672.6	9385.9	-6097.8
									-974.5			
		4				-34904		-64968		698.1		
									-824.2			
4	10		4	215032	95032	12612	504.48	-17452		-126.1	193.6	183.9
									126.1			
		5				60128		30064		0		
									901.9			
5	10		8	210000	-30064	45096	3607.6	15032		901.9	4810.2	-1446.2
									300.6			
		6				30064		0		1202.5		
									-300.6			
6	10		8	210000	-30064	15032	1202.56	-15032		901.9	4810.2	-1446.2
									-901.9			
		1				30064		-30064		0		
		3				30064		0		0		
									0			
7	10		5.7	0	0	30064	1713.65	0		0	0	0
									0			
		6				30064		0		0		
Σ							11334.1					-14720.2
							30064					2453.36

(1)		(2)	(14)	(15)	(16)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
elem	L_e	L_e/t_e	q_{bx}^{**} 10^{-3}	1+4+1 10^{-3}	q_{bx}^{*} 10^{-3}	q_{by}^{**} 10^{-3}	1+4+1 10^{-3}	q_{by}^{*} 10^{-3}	q_{m}^{**} 10^{-3}	1+4+1 10^{-3}	q_{m}^{*} 10^{-3}	q_b KN/mm
			0		35.37	0		35.37	0		32.6	70.74
1	400	40	-30.98	-188.57	4.39	10.03	45.78	45.4	5.14	-7.89	37.74	49.79
			-64.65		-29.28	5.66		41.03	-28.45		4.15	11.75
2	400	40	-92.45	-540.3	-57.08	-4.57	-24.73	30.8	-68.18	-382.59	-35.58	-26.28
			-105.85		-70.48	-12.11		23.26	-81.42		-48.82	-47.22
			-105.85		-23.25	-12.11		70.49	-81.42		-48.82	47.22
3	400	40	-113.4	-683.08	-30.8	-25.51	-167.46	57.09	-68.18	-382.59	-35.58	26.29
			-123.63		-41.03	-53.31		29.29	-28.45		4.15	-11.74
4	400	40	-128	-753.59	-45.4	-86.98	-519.19	-4.38	5.14	-7.89	37.74	-49.78
			-117.96		-35.36	-117.96		-35.36	0		32.6	-70.72
5	800	80	-78.11	-457.9	4.49	-148.42	-832.88	-65.82	-36.76	-196.06	-4.16	-61.33
			-27.5		55.1	-121.24		-38.64	-49.02		-16.42	-16.46
			3.31		38.68	-90.46		-55.09	-49.02		-16.42	-16.46
6	800	80	30.45	125.11	65.82	-39.86	-249.9	-4.49	-36.76	-196.06	-4.16	61.33
			0		35.37	0		35.36	0		32.6	70.73
			0		-47.23	0		-47.23	0		0	94.46
7	565	56.5	3.33	43.32	-43.9	3.33	43.32	-43.9	0	0	0	87.8
			3.33		-17.23	30		-17.23	0		0	34.46
				35.37			35.37			32.6		
				82.6			82.6			32.6		

5. Concluding Remarks

This report has presented the complete mathematical derivations of the equations used for the calculation of cross sectional constants and stress distributions of thin-walled sections. The derivations are based on the same assumptions as Kollbrunner's [7] open section and extend to multi-cell closed sections. These equations can be used for any shape of cross section, both open and closed, and are independent of the orientation of the cross section. A numerical procedure based on these equations has been outlined and verified with several examples. It can be easily implemented into the current SCRAP program.

It should be noted that these equations, which are based on certain assumptions, have some limitations. The major two are that the derivations deal only with prismatic members, and cross sections retain their shape during deformation. Thus, the solutions of the equivalent beam models can only be improved with increasing numbers of beam elements, and are only valid for the unbuckled state of the ship hull's elastic response.

Appendix A - St-Venant Torsion

The governing differential equation of a prismatic beam subjected to St-Venant torsion is

$$T_s = GJ\varphi ' \tag{A1}$$

where J is torsional constant.

For an open thin-walled section which is composed of a number of thin rectangular elements, the torsional constant can be computed as the sum of the values for the individual element.

$$J \approx \sum \frac{1}{3} b_i t_i \tag{A2}$$

where i is the i_{th} element. The shear stress is linearly distributed across the thickness of the wall with a zero average. Its maximum value of shear stress is at the wall surface and can be written as

$$(\tau_s)_i = \frac{T_s t_i}{J} \tag{A3}$$

For a closed thin-walled section with multi-cells, the *torsional constant* is

$$J = 4\{A\}^T[K]^{-1}\{A\} \tag{A4}$$

where A_i is the area of cell i and the entries in matrix [K] are

$$\begin{aligned} k_{ii} &= \sum_{i=1}^{n} \frac{ds}{t} \\ k_{ij} &= -\sum_{ij} \frac{ds}{t} \end{aligned} \tag{A5}$$

The subscript "ii" represents a summation performed along walls of cell 1 and "ij" represents a summation performed along common walls between cell i and j.
For a single cell section, equation (A4) can be reduced to

$$J = \frac{4A^2}{\sum \frac{ds}{t}} \tag{A6}$$

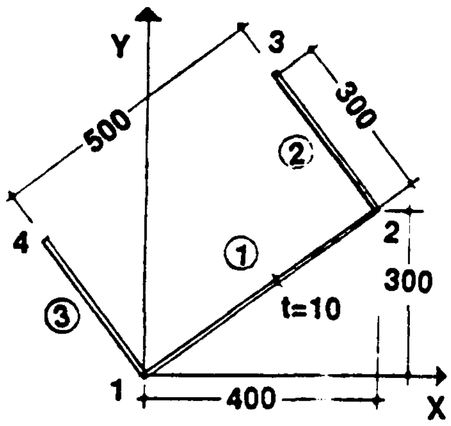
For a beam subjected to a specified torsion T_s, the rate of twisting angle φ' can be obtained from equation (A1). The St-Venant shear flow $(q_s)_i$ in each cell then can be found as

$$\begin{aligned} \{q_s\} &= 2G\varphi'[K]^{-1}\{A\} \\ &= \frac{T_s}{J}[K]^{-1}\{A\} \end{aligned} \tag{A7}$$

Appendix B - Open Section - Example

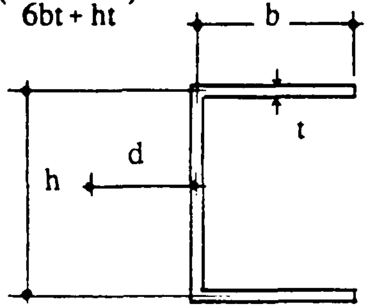
For a channel section with constant wall thickness as shown below, calculate the shear centre and warping constant and verify the results.

The exact solutions of the shear centre and warping constant of a channel section with constant wall thickness are



$$d = \frac{th^2b^2}{4I_x}$$

$$I_\omega = \frac{tb^3h^2}{12}\left(\frac{3bt + 2ht}{6bt + ht}\right)$$



			(1)			(2)		(3)		(4)			(5)	(6)	(8)
elem	t_n	node	X	ΔX	L_n	A_n 10^3	X_nA_n 10^5	x	ΔS_x 10^5	S_x	1+4+1 10^5	I_x 10^7	I_{xy} 10^7	$\Delta\Omega_n$	$I_{x\Omega}$ 10^{10}
		1	0					-151		-7.23					
									-1.28						
1	10		200	400	500	5	10	49		-8.51	-46.05	-184.2	-138.2	40700	-18.74
									3.73						
		2	400					249		-4.78					
									3.06						
2	10		310	-180	300	3	9.3	159		-1.72	-11.66	20.98	-27.98	75060	-8.75
									1.71						
		3	220					69		0					
		4	-180					-331		0					
									-4.29						
3	20		-90	180	300	3	-2.7	-241		-4.29	-24.39	-43.9	58.54	74940	-18.28
									2.94						
		1	0					-151		-7.23					
Σ						11	16.6					-207.22	-107.		-45.17
							151				-Σ/6	34.54	17.93		7.63

$X_c = 151; \quad Y_c = 215$

$x_p = 118.85; \quad y_p = -160.16$

$X_p = 269.85; \quad Y_p = 54.85$

			(1)			(2)		(3)		(4)			(5)	(8)
elem	t_k	node	Y	ΔY	L_k	A_k 10^3	$Y_k A_k$ 10^5	y	ΔS_Y 10^5	S_Y	1+4+1 10^5	I_Y 10^7	I_{YX} 10^7	$I_{Y\omega}$ 10^{10}
		1	0					-215		-2.86				
									-3.5					
1	10		150	300	500	5	7.5	-65		-6.36	-34.41	-103.23	-137.64	-14
									0.25					
		2	300					85		-6.11				
									2.18					
2	10		420	240	300	3	12.6	205		-3.93	-21.83	-52.39	39.29	-16.38
									3.98					
		3	540					325		0				
		4	240					25		0				
									-0.53					
3	20		120	-240	300	3	3.6	-95		-0.53	-4.98	11.95	-8.96	-3.73
									-2.33					
		1	0					-215		-2.86				
Σ						11	23.7					-143.63	-107.31	-34.11
							215				-Σ/6	23.95	17.88	5.69

		(1)	(2)	(9)		(10)	(11)		(12)		(13)
elem	t_k	node	A_k 10^3	$\Delta\Omega_{jk}$	Ω_i-Ω_j	Ω_o	ω	ΔS_ω 10^5	S_ω 10^5	1+4+1 10^5	I_ω 10^{10}
		1			74635		29360		-244		
								367			
1	10		5	-59019		2256	0		123	4	-2.36
								-367			
		2			15616		-29360		-244		
								-159			
2	10		3	75365		1599	8140		-403	-1856	-1392
								403			
		3			90981		45640		0		
		4			0		-45640		0		
								-403			
3	10		3	74635		1120	-8140		-403	-1856	-1392
								159			
		1			74635		29360		-244		
						4975					-2786.36
						45640					464.4

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The equivalent beam model is widely used for predicting strength and vibration of a ship hull in a preliminary analysis. It can also be used for checking results in a large finite element model and for parametric studies of ship behaviour. This method treats a ship hull as a series of prismatic segments connected together. Each segment has its sectional properties, real and virtual masses. The program SCRAP was developed at Defence Research Establishment Atlantic for calculation of cross sectional constants and estimation of mass properties. It prepares input data files for the finite element programs VAST and TORSON and interprets the analytical results. SCRAP can be only used for some specific sections at the present time. For arbitrarily oriented sections it may give an incorrect shear centre and warping constant and thus the wrong stress distributions.

This report presents the mathematical derivations of the equations used for the calculation of cross sectional constants and stress distributions of thin-walled sections. As an improvement over the current SCRAP program, these equations are applicable to any shape of cross section, both open and closed, and are independent of the orientation of the cross section. A computer-oriented step-by-step procedure based on these equations is outlined. Several examples are also presented to verify the procedure.

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